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THE RELATION BETWEEN THE ELASTIC STRENGTHS OF STEEL IN TENSION, COMPRESSION, AND SHEAR

BY
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BULLETIN No. 115

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BULLETIN NO. 115 NOVEMBER, 1919

THE RELATION BETWEEN THE ELASTIC STRENGTHS OF STEEL IN TENSION, COMPRESSION, AND SHEAR

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THE RELATION BETWEEN THE ELASTIC STRENGTHS OF STEEL IN TENSION, COMPRESSION, AND SHEAR

I. Introduction

1. *Preliminary.*—This bulletin presents the results of experiments with six grades of steel, three carbon steels and three alloy steels; namely, soft, mild, and medium carbon steel; and vanadium, nickel, and chrome-nickel alloy steel. The elastic strength in tension, in compression, and in shear is given for each of the six grades of steel. The elastic strength in shear is found from tests in torsion with solid cylindrical specimens and with thin-walled hollow cylindrical specimens. Furthermore, a factor is found by the use of which the true or correct shearing elastic strength may be calculated from the elastic strength obtained from a test of a solid specimen. The ratio of this true elastic shearing strength to the elastic tensile strength is given for each grade of steel and its bearing on the theory of combined stress is discussed. The ratio of the elastic tensile strength to the elastic compressive strength is also given and the effect of the amount of rolling upon the elastic tensile and compressive strengths is discussed. The effect of the direction of rolling upon all three elastic strengths is considered for one of the materials; namely, nickel steel.

Our knowledge of the breakdown of elastic action of ductile materials, particularly in the case of combined loading, is far from complete. The various theories of combined stress lead to results which differ rather widely when applied to various machine parts or to structural elements, such as thick cylinders, flat plates, crank shafts, webs of girders, etc. The maximum shear theory of combined stress for ductile materials as expressed by Guest's law, which has gained rather wide acceptance in recent years, assumes that the elastic shearing strength is one-half of the elastic tensile strength. Available experimental results, however, have, in general, failed to justify this assumption. The importance of the limitation imposed by the shearing stress upon the elastic strength of ductile material is, of course, generally recognized. If the maximum strain theory holds until the shearing yield point is reached, as is indicated in recent tests,* it is of

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***See Bulletin No. 85, Engineering Experiment Station, University of Illinois, "Strength and Stiffness of Steel under Biaxial Loading," by A. J. Becker.**

special importance to know the relation between the shearing and the tensile (and compressive) elastic strengths for various grades of ductile and semi-ductile steels. The main object of the investigation herein recorded was to determine carefully the elastic shearing strength of ductile and semi-ductile steel and to find the ratio of the elastic shearing strength to the elastic tensile strength with the hope that definite information would thereby be obtained on the breakdown of the elastic action of various grades of steel and on the limits of the theories of combined stress.

Apart from the problem of combined stress there has been also a lack of knowledge of the correct elastic shearing strength of various grades of steel and of the general nature of elastic shearing failure as well as of methods of determining the correct shearing strength from tests.

The facts brought out in connection with the elastic compressive strengths of the various materials tested should also add to our knowledge of the elastic behavior of steel and it is felt that questions are raised which may become of considerable importance. There is some evidence indicating that the amount of rolling (roughly indicated by the thickness of the rolled material) may become an important factor in the selection of the proper criterion of elastic strength as well as in estimating the elastic compressive and shearing strengths from a tension test. This question may be of considerable importance in connection with compression members and with certain cases of combined stress. It may also have an important bearing on the problem of the fatigue of steel under repeated stress.

The severe uses under the many and varied new conditions, such as have arisen during the war, and the development of new requirements for machine parts have brought out the need for fuller information on the physical properties of carbon and alloy steel. This bulletin is presented as a contribution toward filling this need.

2. *Acknowledgment.*—All of the experimenting was done in the Laboratory of Applied Mechanics of the University of Illinois. Acknowledgment is made to Professors A. N. Talbot and H. F. Moore for the interest shown and helpful suggestions offered during the investigation. Some preliminary experimenting had been done by Professor Moore to determine the merits of various forms of shear specimens. This work was found to be of considerable value in planning certain parts of the investigation herein described.

II. MATERIALS, TEST SPECIMENS, AND METHOD OF TESTING

3. *Materials.*—As already stated, six grades of steel were tested; namely, soft, mild, and medium carbon steel, and vanadium, nickel, and chrome-nickel alloy steel. Chemical analyses were made of the nickel steel and the chrome-nickel steel only. All of the material except the nickel and the chrome-nickel steel was bought in the open market. The nickel steel specimens were cut from one of the ends of the untested riveted-joint test specimens made for the Board of Engineers of the Quebec Bridge. (The other riveted-joint specimens were tested at the University of Illinois and a report of the tests was made in Bulletin No. 49 of the Engineering Experiment Station.) The chrome-nickel steel specimens were made from ¾-inch square bars of the same material as that used in the chrome-nickel steel riveted-joint specimens also described in Bulletin No. 49 referred to above. The chemical analyses of these two alloy steels as reported in Bulletin No. 49 are given in Table I.

TABLE 1

CHEMICAL COMPOSITION OF NICKEL AND CHROME-NICKEL STEEL

Element	Nickel Steel Per Cent	Chrome-Nickel Steel Per Cent
Carbon	0.258	0.191
Sulphur	0.008	0.035
Phosphorus	0.044	0.042
Manganese	0.700	0.485
Nickel	3.330	0.733
Chromium	.....	0.170

All of the specimens of chrome-nickel steel did not come from the same bar. Three different bars rolled from the same heat were used. The medium steel specimens were made from two bars supposed to be of the same material and billed as 40-point carbon steel. The specimens of each of the other four materials were made from the same bar or piece. All of the material was hot-rolled only. No cold-rolled material was used. The specimens of soft steel came from a bar 3½ inches in diameter by 20 feet in length. The mild steel specimens and also the vanadium steel specimens were cut from a bar ⅞ inch in diameter by 16 feet in length; the nickel steel specimens were cut from a slab 2 inches in thickness by 7½ inches in width. The large soft steel bar (3½ inches in diameter) was used in order to obtain large torsion specimens as well as small ones.

Although the chemical analysis of all of the materials tested can not be given, the range of material used is well indicated by the elastic tensile strengths as given in Table 2. While some of the material used did not have a well defined yield point, yet all six grades of steel are considered to be ductile or semi-ductile material since the tensile fractured area in all cases showed a considerable reduction.

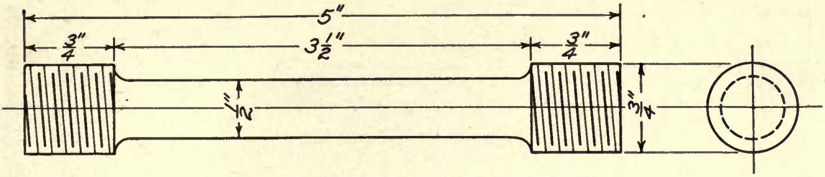
TABLE 2

ELASTIC TENSILE STRENGTHS OF MATERIALS USED

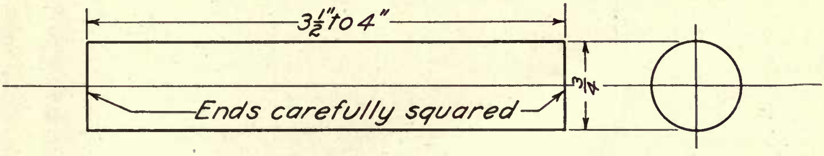
	Materials	Elastic Strength* lb. per sq. in.
Carbon Steel	Soft	21 000
	Mild	32 800
	Medium	45 000
Alloy Steel	Vanadium	52 500
	Nickel	38 000
	Chrome-nickel	35 300

*Proportional limit (see Fig. 5) is here used as a measure of the elastic strength.

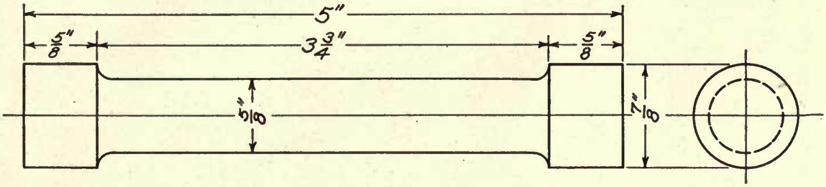
4. *Test Specimens.*—Tension, compression, and shear (torsion) specimens were made from each of the six materials. Both solid and hollow cylinders were used for the torsion specimens; hollow specimens of three different wall thicknesses were tested. In general from three to nine specimens of each type were tested for each material. The total number of specimens tested was 160, exclusive of a considerable number used in preliminary tests in perfecting the measuring apparatus. In obtaining specimens from a bar, care was taken to cut the specimens in rotation so as to avoid the effects of any systematic variation in the properties of the material along the bar. In the case of nickel steel, specimens were cut from a slab 2 inches thick by 7½ inches wide in such a way that the longitudinal axes of some of the specimens were parallel to the direction in which the slab was rolled, while the longitudinal axes of other specimens were perpendicular to the direction of rolling. Large torsion specimens both solid and hollow, as well as small ones, were made from the 3½-inch bar of soft steel.



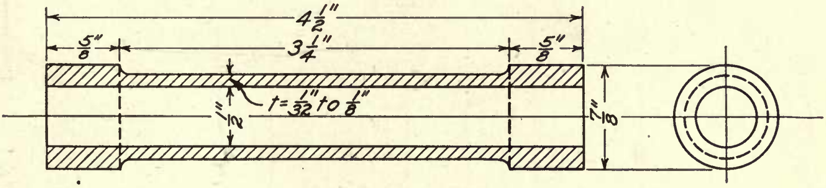
Tension Specimen



Compression Specimen



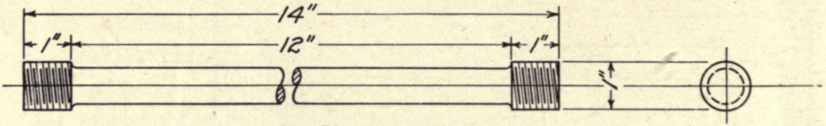
Solid Torsion Specimen



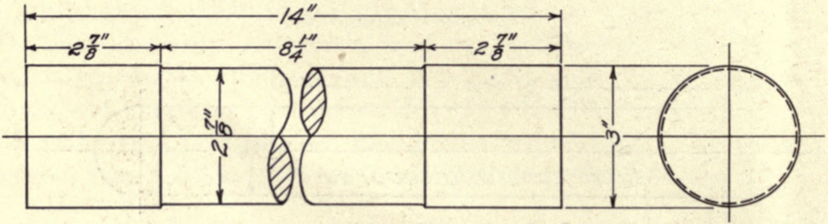
Hollow Torsion Specimen

FIG. 1. FORM AND DIMENSIONS OF SMALL TEST SPECIMENS

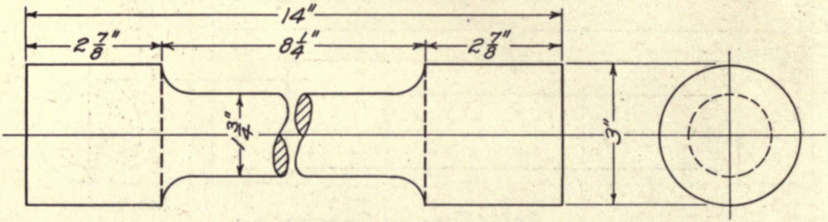
Fig. 1 gives the dimensions of the small specimens used for all the materials and Fig. 2 gives the dimensions of the large specimens of soft steel. Fig 3 shows some of the specimens both before and after testing and also the apparatus used for measuring the thickness of the walls of the small hollow specimens. The length of the small hollow specimens was limited by the length of the ½-inch hole which could be drilled through the specimens from one end. The large hollow specimens were first drilled from both ends and then bored out to the



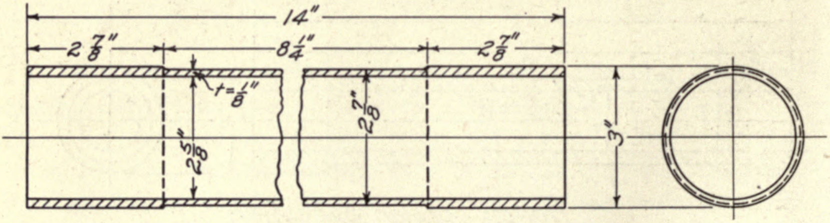
(a), Tension Specimen



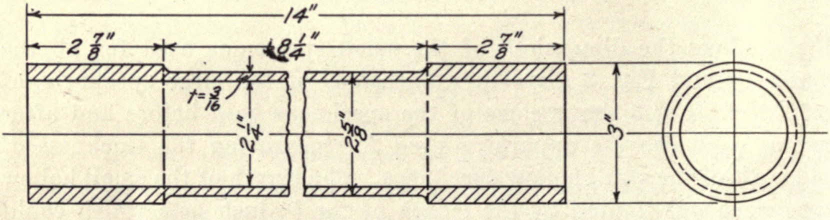
(b), Large Solid Torsion Specimen



(c), Medium Solid Torsion Specimen



(d), Large Hollow Torsion Specimen



(e), Medium Hollow Torsion Specimen

FIG. 2. FORM AND DIMENSIONS OF LARGE TEST SPECIMENS

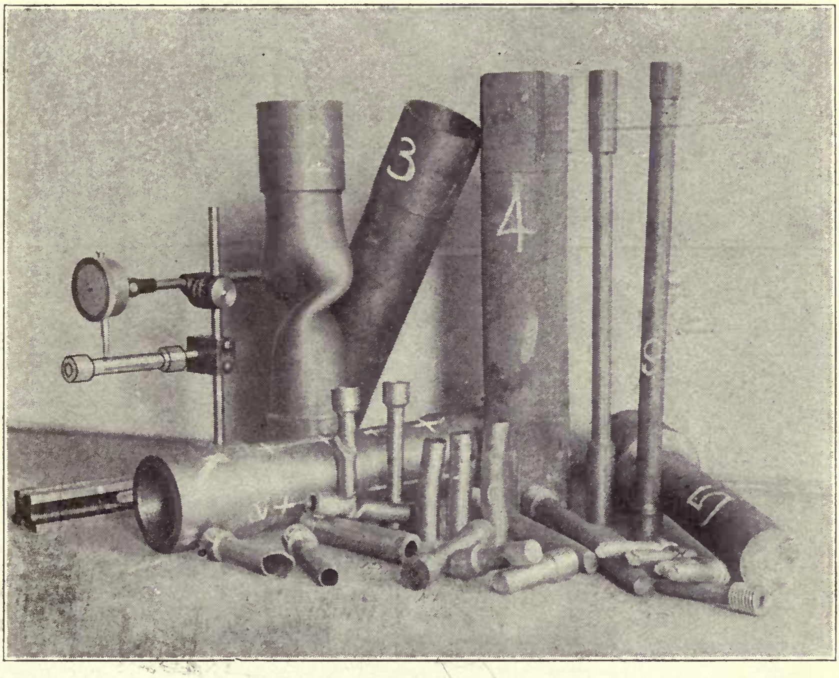


FIG. 3. VIEW OF SOME OF THE SPECIMENS

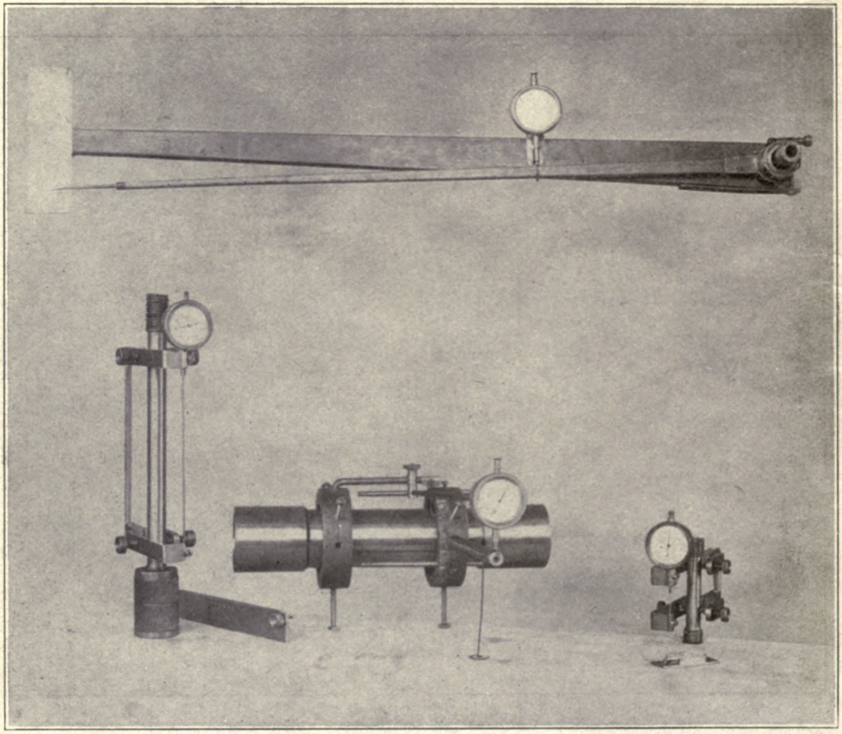


FIG. 4. VIEW OF DEFORMATION MEASURING APPARATUS

desired inner diameter. After the desired inner diameter was obtained, each specimen, whether small or large, was mounted on a mandrel and the outside diameter turned down to the required dimension. The wall thickness of the hollow specimens was measured at four points, 90° apart, around each of four sections along the length of the specimens. The outside diameters at each of the four sections were also taken and the smallest cross section was used in the calculations. The wall thickness could be read to 0.0001 inch. In the case of the large hollow specimens the wall thickness was found in the same way although larger apparatus was required.

5. *Tension Tests.*—The tension test specimens were made with threaded ends and with a diameter of ½ inch (see Figs. 1 and 2). A 2-inch gage length was used with all the specimens except those of soft steel with which a gage length of 8 inches was used. The long specimens of soft steel were used in order to determine the modulus of elasticity more accurately than could be done with specimens having a 2-inch gage length. The modulus of elasticity or stiffness of the various materials, however, is not discussed in this bulletin.

All of the tension tests were made in a 100 000-pound Riehle universal testing machine which had been calibrated over the range used in these experiments. Spherical seated holders were used in all of the experiments.

The extensometers used are shown in Fig. 4. The unit-elongation could be read directly to 0.00025 inch per inch when using a 1/1000-inch Ames dial for a 2-inch gage length, or to 0.000025 inch per inch when using a 1/10 000-inch Ames dial. Both dials were used but it was found that there was little advantage in using the more sensitive dial. The unit-elongation could be estimated to one-tenth of the above values.

6. *Compression Tests.*—The compression test specimens in nearly all cases were made with a diameter of ¾ inch and with a length of 3½ inches to 4 inches as shown in Figs. 1 and 3. A gage length of 2 inches was used in all cases and the deformation measuring apparatus was the same as that used in the tension tests. Care was taken to square off the ends of the specimens in the lathe so that they were smooth and perpendicular to the axis of the specimens. Each specimen was carefully centered in a 100000-lb. Riehle universal testing machine by means of a templet; spherical seated bearing blocks were

used. The testing machine had been calibrated over the range used in these experiments and for a considerable number of tests it was the same machine as was used for the tension tests. The length of specimen used (3½ inches to 4 inches) corresponds to a slenderness ratio value of 19 to 21. In most cases the specimen failed as a flat-ended column (see Fig. 3). It is felt that with the very careful centering the real elastic strength of the material was developed in all cases although the ultimate compressive strength showed considerable variation.

7. *Torsion Tests.*—After some preliminary study and experimenting, a torsion test of a cylindrical specimen was decided upon as being the most satisfactory means for determining the elastic shearing strength of the material. Both solid and hollow cylindrical specimens were used. The small hollow specimen (see Figs. 1 and 3) were made with three different dimensions of the wall thickness for several of the materials; namely, 1/32 inch, 1/16 inch, and 1/ 8 inch (approximate dimensions), the nominal inside diameter being ½ inch for each specimen. A wall thickness of 1/32 inch makes it possible to determine, very closely, the true elastic shearing strength of the material.

In the case of soft steel, large solid and hollow specimens were made from the 3½-inch bar in addition to the small specimens already described. These large specimens were 14 inches long with a gage length of 5 inches and with a length of 8 inches between shoulders. The diameter, between shoulders, of the first large specimens made was 2⅞ inches for both the solid and the hollow specimens. From the results of the tests, however, it was at first thought that the specimens were not long enough; hence one solid specimen was made 20 inches long and tested, but the results showed no effect due to the changed length. Three solid specimens were then made, 1¾ inches in diameter, for which the load on the machine at the yield point would be about the same as for the hollow specimens. The effect of this change is discussed later. The large hollow specimens were made in two sizes; namely, with a wall thickness of ⅛ inch and an outside diameter of 2⅞ inches and with a wall thickness of 3/16 inch and an outside diameter of 2⅝ inches, as shown in Figs. 2 and 3. In testing the former, plugs had to be fitted into the ends to keep the ends from collapsing in the grips, while in testing the latter this method was not necessary. The ratio of the wall thickness of the hollow specimens to the diameter for the thinner walled specimens is about the same for the large specimens as for the small ones, the value of the ratio being approximately 1 to 20.

All of the large specimens were tested in a 230 000-inch-pound Olsen torsion testing machine, while the small specimens were tested in a Riehle hand power pendulum torsion testing machine. A special light pendulum was used in the Riehle machine for the hollow thin-walled specimens and special apparatus was made for measuring the load. Both torsion machines were calibrated carefully.

All of the small specimens from the 3½-inch bar of soft steel were made by first cutting a 14-inch length of the bar longitudinally into quarters and turning the small specimens from one or more of the quarters.

The apparatus for measuring the deformation in the torsion tests is shown in Fig. 4. Both a 1/1000-inch and a 1/10 000-inch dial were used on each apparatus, so that a considerable range in sensitiveness was obtained to suit the variations in wall thickness, etc.

III. Experimental Data and Discussion

8. *Criteria of Elastic Strength.*—Perhaps the best indication of the elastic strength of a material is the elastic limit; that is, the greatest unit-stress which the material can resist without taking a permanent set. The process of determining the elastic limit, however, is so long that it was considered impracticable for the purposes of this investigation.

For the purpose of the comparison of the elastic strengths of various materials or of the same material under different types of stress any one of several unit-stresses as represented on the stress-strain curve may be used. Three such points on the stress-strain curve are used in this investigation; namely, the proportional limit (sometimes called proportional elastic limit), the yield point, and a point between the proportional limit and the yield point called the semi-elastic point or the useful limit point. It is assumed that elastic action only takes place until the proportional limit is reached, while plastic action only occurs at the yield point. Any point on the stress-strain curve between the proportional limit and the yield point corresponds to an action in the material which is partly elastic and partly plastic. Several arbitrary methods have been proposed for conveniently locating such a point. The semi-elastic point or useful limit point used in this investigation is defined as the unit-stress at which the rate of deformation is 100 per cent greater than at zero-stress. This point was used by the Committee of the American Society of Civil Engineers in the analysis* of the tests of large built-up-columns and is very similar to Johnson's apparent elastic limit.† In Fig. 5 the point U represents the useful limit point which is found by first laying off KN equal to two times KM and then drawing a line parallel to ON tangent to the stress-strain curve, the point of tangency being U. Fig. 5 also shows the proportional limit and yield point.

In the study of the elastic failure of the materials tested, the three criteria of elastic strength mentioned above (porportional limit, useful limit point, and yield point) taken together furnish a safer guide than any one alone. The elastic strengths of the materials tested as

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*Proc. A. S. C. E. Dec., 1917.

†Johnson: The Materials of Construction, p. 10, 1918.

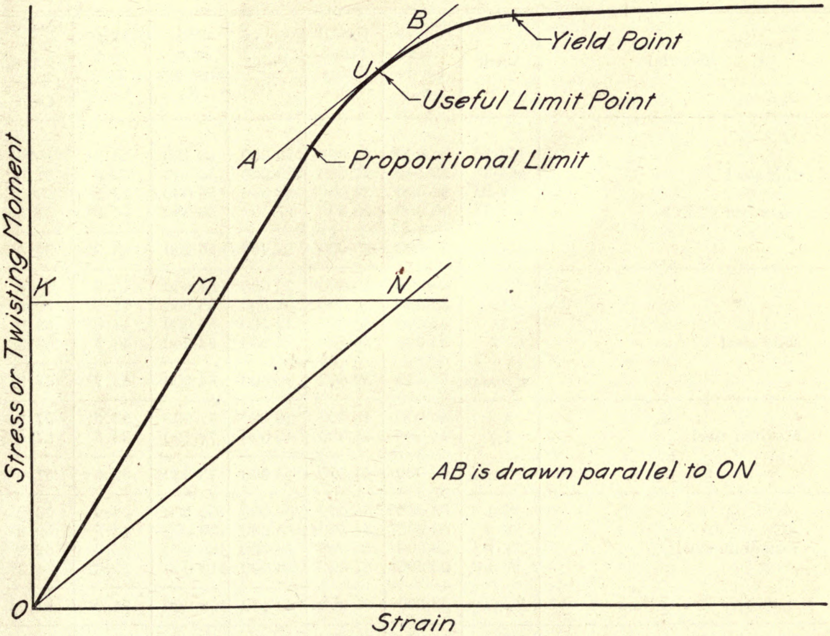

FIG. 5. TYPICAL STRESS-STRAIN DIAGRAM SHOWING THE USEFUL LIMIT POINT

indicated by each of these three criteria for tension, for compression, and for shear are given in Tables 3, 4, and 5 for the individual specimens while the averages are given in Table 6.

9. *Shearing Strengths.*—As has already been stated, the elastic strength in shear was found from tests of solid cylindrical torsion specimens and from thin-walled hollow cylindrical specimens.

It has been commonly recognized that the yield point in shear as found from the test of a solid cylindrical torsion specimen does not represent the correct shearing yield point of the material since all of

TABLE 3

RESULTS OF TENSION TESTS

Gage length 2 in. Diameter ½ in. except as noted. Stress in lb. per sq. in.

Material	Mark	Proportional Limit S_p	Useful Limit Point S_e	Yield Point S_y	Ultimate Strength S_u	Elongation Per Cent	Reduction of Area Per Cent
Soft steel (diameter ¾ in.)	M3-1d	21 000	24 000	29 000	56 100	30.0*	50.6
	M3-T	22 500	27 000	29 000	53 100	50.0	57.6
	M3-2d	20 500	24 000	28 000	56 000	27.0*	53.5
	M3-3d	20 000	25 000	27 500	56 200	29.8*	55.5
	Average	21 000	25 000	28 400	55 400	28.9*	54.3
Mild steel	3	33 000	33 000	33 000	54 900	45.3	68.8
	11	33 000	33 000	35 500	55 500	42.0	70.0
	15	32 000	32 000	32 500	55 000	44.0	69.0
	7	33 000	33 000	33 500	54 300	43.5	63.8
	Average	32 800	32 800	33 600	54 900	43.7	67.9
Medium steel	4-3	46 000	46 000	46 500	77 900	33.5	57.6
	4-7	44 000	44 000	45 000	78 100	33.5	57.6
	Average	45 000	45 000	45 800	78 000	33.5	57.6
Vanadium steel	V-1	53 500	56 000	60 500	109 500	25.0	50.8
	V-3	53 000	54 500	59 000	108 000	23.5	50.8
	V-7	50 000	56 000	59 000	108 000	23.5	52.0
	V-15	53 500	57 000	63 000	110 000	15.6†	34.5†
	Average	52 500	55 900	60 400	109 000	24.0	51.2
Nickel steel Stress parallel to direction of rolling	P-1	35 500	39 000	47 000	87 800	28.5	57.5
	P-2	38 500	40 000	47 000	88 000	28.0	55.2
	Average	37 000	39 500	47 000	87 900	28.3	56.4
Nickel steel Stress perpendicular to direction of rolling	C-1	39 000	40 500	47 000	86 500	17.5	36.0
	C-2	41 000	41 500	48 000	88 100	24.0	51.2
	C2-1d	36 500	40 000	48 000	88 500	22.5	30.0†
	Average	38 800	40 700	47 700	87 700	21.3	39.0
Chrome-nickel steel	L-3	33 500	35 000	38 000	64 500	39.5	62.8
	5-C	30 500	32 500	35 500	67 500	37.5	58.8
	L-5	38 000	38 000	38 500	66 700	38.5	60.5
	8-C	28 000	30 500	36 000	65 500	34.0	64.0
	L8-1d	33 000	35 000	37 500	64 500	37.5	61.2
	L8-2d	36 500	36 500	38 000	65 000	38.0	61.6
	Average	33 300	34 600	37 300	65 600	37.5	61.5

*Gage length 8 in.

†Broke in punch mark.

TABLE 4

RESULTS OF COMPRESSION TESTS

Gage length 2 in. Approximate diameter ¾ in. Length of specimen 3½ in. to 4 in. Stress in lb. per sq. in.

Material	Mark	Proportional Limit S_p	Useful Limit Point S_e	Yield Point S_y	Ultimate Strength S_u
Soft steel	M-C	25 500	25 500	26 200	46 400
	M-10	29 000	29 000	29 000	54 000
	M-S	26 000	26 000	27 200	45 000
	M-11	27 000	27 500	29 000	54 200
	M-T	26 500	26 500	27 000	47 400
	M-12	27 000	27 000	28 500	70 000
	Average	26 800	26 800	27 800	52 800
Mild steel	4	36 000	36 000	37 000	56 000
	8	36 000	36 000	36 500	61 000
	12	32 000	32 500	35 000	54 200
	16	35 000	35 000	35 500	58 700
	Average	34 800	34 900	36 000	57 500
Medium steel	4-4	37 000	40 000	46 000	80 800
	4-8	45 500	45 500	47 000	82 300
	Average	41 300	42 800	46 500	81 600
Vanadium steel	V-4	56 000	57 500	64 000	103 500
	V-8	56 000	58 500	64 000	111 500
	V-12	56 000	58 500	65 000	96 500
	V-1-0	52 500	55 000	64 000	106 000
	V-16	51 000	57 000	63 500	87 000
	Average	54 300	57 300	64 100	101 000
Nickel steel. Stress parallel to direction of rolling	P-3	42 000	46 000	49 000	65 200*
	P-4	42 500	47 000	50 500	99 500
	P-C	36 000	40 000	48 500	79 000
	P-D	37 000	38 000	48 500	86 600
	Average	39 400	42 800	49 100	88 400
Nickel steel Stress perpendicular to direction of rolling	C-A	38 500	43 000	47 500	89 500
	C-B	38 500	41 500	48 000	95 200
	C-4	38 500	40 500	50 000	92 500
	C-5	37 500	40 000	50 000	71 000†
	C-6	40 500	41 500	50 000	61 000†
	Average	38 700	41 300	49 100	92 400
Chrome-nickel steel	L-3	36 500	37 500	41 000	63 400
	L5-1d	39 000	39 000	40 000	70 200
	5-E	37 600	37 600	38 500	......
	L-8	35 500	36 500	39 000	65 700†
	8-H	37 000	37 000	38 500	......
	L5-2d	40 000	40 000	40 500	57 800†
	Average	37 600	37 900	39 600	64 300

*One end crushed. † Both ends crushed.

TABLE 5

RESULTS OF TORSION TESTS

Stress in lb. per sq. in. d = approximate outside diameter in inches
t = approximate wall thickness in inches.
l = gage length in inches.

Material	Type of Specimen	Mark	Proportional Limit S_p	Useful Limit Point S_e	Yield Point S_y	Modulus of Rupture S_r
Soft steel from 3½-in. bar.	Small solid	M-30*	15 400	15 400	17 400	......
		M-3-0	15 400	18 300	19 900	......
	$d=5/8$	M-1-D	14 100	14 900	16 800	......
		M-2-D	13 900	15 400	18 500	......
	$l=2$	M-3-D	14 600	14 600	15 800	......
		31	14 000	16 500	18 600	60 500
		33	13 600	16 000	18 500	62 500
		34	14 400	17 700	19 000	61 500
		Average	14 400	16 100	18 100	61 500
	Small hollow	MS-1	12 300	13 600	19 200‡	......
		MS-2	11 600	13 400	19 200‡	......
	$d=5/8$	MS-3	10 300	12 700	17 000‡	......
	$t=1/32$	35	10 700	15 500	21 100‡	30 200¶
		37	11 600	17 000	23 400‡	....
	$l=2$	38	13 400	16 900	21 800‡	32 000¶
		Average	11 700	14 900	20 300‡	......
	Large solid	M-1-1	12 300	14 500	20 200	......
		M-4-3	12 900	14 800	20 500	......
	$d=2\frac{7}{8}$	M-6-2	13 200	15 500	20 200	......
		M-S-10	14 000	14 400	20 400	......
	$l=5$	M-E-0	18 600	19 400	20 800	......
		Average	14 200†	15 700†	20 400	......
	Large hollow	M-2-1	14 300	16 700	18 500	......
	$d=2\frac{7}{8}$	M-5-2	14 200	16 100	18 100	......
	$t=1/8$	M-7-3	14 300	16 100	17 800	......
	$l=5$	Average	14 300	16 300	18 100	......
	Medium solid	MM	17 500	18 500	19 300	59 200
	$d=1\frac{3}{4}$	MM-0	16 500	18 200	19 100	......
	$l=5$	ME-0-1	16 000	17 700	18 900	......
		Average	16 700	18 100	19 100	......
	Medium hollow	MM-1	15 900	16 800	17 800	42 000¶
	$d=2\frac{5}{8}$	MM-2	16 000	17 600	18 500	......
	$t=3/16$	MM-3	17 600	18 400	19 200	......
	$l=5$	Average	16 500	17 600	18 500	......

*Gage length 8 in.

†Results in error as explained in Section 9.

‡Yield point not well defined

¶Collapsed.

TABLE 5—(CONTINUED)

RESULTS OF TORSION TESTS

Stress in lb. per sq. in. d = approximate outside diameter in inches.
t = approximate wall thickness in inches.
l = gage length in inches.

Material	Type of Specimen	Mark	Proportional Limit S_p	Useful Limit Point S_e	Yield Point S_y	Modulus of Rupture S_r
Mild steel from $\frac{7}{8}$-in. bar Gage length 2 in.	Solid $d=\frac{5}{8}$	14	22 700	22 700	23 800	63 400
		2	22 200	22 200	24 200	60 200
		6	24 200	24 200	24 200	......
		10	22 300	22 300	23 300	......
		Average	22 900	22 900	23 900	61 800
	Hollow $d=0.8$ $t=\frac{1}{8}$	H	21 000	21 000	21 000	......
		K	21 000	21 000	21 000	......
		L	20 900	20 900	20 900	57 000
		N	21 200	21 200	21 200	57 200
		Average	21 000	21 000	21 000	57 100
	Hollow $d=\frac{5}{8}$ $t=\frac{1}{16}$	1	22 600	22 600	23 600	47 600*
		5	20 500	20 500	22 100	......
		9	20 600	20 600	22 900	......
		13	21 600	21 600	23 000	......
		Average	21 300	21 300	22 900	......
	Hollow $d=0.56$ $t=\frac{1}{32}$	A	20 000	20 000	21 800	55 000†
		B	20 100	20 100	21 400	33 000*
		C	19 200	19 200	20 500	......
		D	19 800	19 800	20 500	......
		E	19 400	19 400	21 200	......
		Average	19 700	19 700	21 100	......
Medium steel Gage length 2 in.	Solid $d=\frac{5}{8}$	4-2	30 900	31 600	33 800	79 300
		4-6	31 800	31 800	33 800	......
		Average	31 400	31 700	33 800	......
	Hollow $d=\frac{5}{8}$ $t=\frac{1}{16}$	4-1	27 100	27 600	29 300	56 500*
		4-5	26 600	27 400	29 700	69 200†
		Average	26 900	27 500	29 500	......

*Collapsed. †Sheared, rod filled center.

TABLE 5—(CONTINUED)

RESULTS OF TORSION TESTS

Stress in lb. per sq. in. d = approximate outside diameter in inches.
t = approximate wall thickness in inches
l = gage length in inches.

Material	Type of Specimen	Mark	Proportional Limit S_p	Useful Limit Point S_e	Yield Point S_y	Modulus of Rupture S_r
Vanadium steel from ⅞-in. bar Gage length 2 in.	Solid	V-2	32 400	36 000	42 600	......
		V-10	32 200	33 800	43 000	......
	$d=5/8$	V-6	35 600	37 600	44 000	109 000
		V-14	34 200	37 300	45 000	......
		V-1-V	37 700	39 400	46 500	......
		V-2-V	35 100	39 200	46 500	102 000
		Average	34 500	37 200	44 600	105 800
	Hollow	V-B	37 200	37 200	43 000	......
		V-C	36 200	38 100	42 500	......
	$d=0.8$	V-20	39 200	39 200	43 000	......
		V-21	37 200	38 000	42 000	......
	$t=1/8$	V-22	37 700	37 700	42 500	96 000
		Average	37 500	38 000	42 600	......
	Hollow	V-1	36 000	38 000	43 000	......
	$d=5/8$	V-5	32 800	35 000	43 500	74 300
		V-9	27 800	33 000	41 500	72 800*
	$t=1/16$	V-13	31 800	34 000	42 000	161 000†
		Average	32 100	35 000	42 500‡	......
	Hollow	V-D	28 300	28 300	44 500	......
		V-E	28 000	28 000	40 500	......
	$d=0.56$	V-23	26 600	30 400	42 500	72 600†
		V-24	32 000	37 000	50 000	78 000†
	$t=1/32$	V-25	28 200	31 400	42 700	73 200†
		V-3-V	28 400	34 000	39 800	......
		V-4-V	23 900	29 100	37 400	......
		V-5-V	25 500	29 500	37 600	......
		Average	27 600	30 900	41 900‡	......

*Collapsed. †Sheared, rod filled center. ‡ Yield point not well defined.

TABLE 5—(CONTINUED)

RESULTS OF TORSION TESTS

Stress in lb. per sq. in. d = approximate outside diameter in inches.
t = approximate wall thickness in inches.
l = gage length in inches.

Material	Type of Specimen		Mark	Proportional Limit S_p	Useful Limit Point S_e	Yield Point S_y	Modulus of Rupture S_r
Nickel steel from slab 2 in. thick. Gage length 2 in.	Stress parallel to direction of rolling	Solid	C-1	22 800	25 800	36 400	72 000
			C-2	25 700	27 400	36 800	......
		$d = \frac{5}{8}$	C-3	26 000	27 600	......	77 500
			Average	24 800	26 900	36 600	74 800
		Hollow	C-4	27 000	27 000	35 800	64 000†
		$d = \frac{5}{8}$	C-5	23 500	23 500	34 400	......
		$t = \frac{1}{16}$	C-6	22 300	26 700	35 300	73 500‡
			Average	24 300	25 700	35 200*	......
		Hollow	C-5d	21 000	23 300	34 200	......
			C-6d	21 500	24 800	34 200	......
		$d = 0.56$	C-1d	23 900	24 400	34 800	......
			C-3d	22 300	24 000	36 200	......
		$t = \frac{1}{32}$	C-4d	21 000	23 500	35 500	......
			C-2d	22 300	25 700	36 000	......
			Average	22 000	24 300	35 200*	......
	Stress perpendicular to direction of rolling	Solid	P-4	26 000	28 000	36 500	77 100
		$d = \frac{5}{8}$	P-5	26 000	30 000	40 500	......
				26 000	29 000	38 500*	......
		Hollow	P-6	24 400	28 200	41 500	75 000‡
		$d = \frac{5}{8}$	P-7	24 200	27 200	37 000	76 000
		$t = \frac{1}{16}$	Average	24 300	27 700	39 200*	......
		Hollow	P-8	Spoiled in	testing		
		$d = 0.56$	P-9	20 900	23 700	32 100	......
		$t = \frac{1}{32}$	P-10	20 600	23 400	31 500	......
			P-11	20 100	23 400	32 200	......
			Average	20 500	23 500	32 000*	......
Chrome-nickel steel. Gage length 2 in.		Solid	L3-2d	24 100	24 700	26 300	......
			L3-1d	22 800	24 500	27 900	......
		$d = \frac{5}{8}$	L-5	24 100	25 200	28 000	62 200
			L-8	25 200	26 200	27 200	......
			Average	24 100	25 200	27 400	......
		Hollow	L8	23 800	23 800	24 800	51 200†
		$d = \frac{5}{8}$	L5	22 300	22 300	23 600	......
		$t = \frac{1}{16}$	L3	22 300	22 300	24 300	51 600†
			Average	22 800	22 800	24 200	......

*Yield point not well defined.

† Collapsed.

‡Sheared, rod filled center.

TABLE 6

AVERAGES OF THE STRENGTHS OBTAINED FROM THE INDIVIDUAL SPECIMENS IN TENSION, IN COMPRESSION, AND IN SHEAR FOR EACH GRADE OF STEEL TESTED

Stress in lb. per sq. in.

Material	Kind of Stress		Number of Tests	Approximate Outside Diameter Inches	Gage Length Inches	Nominal Wall Thickness Inches	Proportional Limit S_p	Useful Limit Point S_e	Yield Point S_y	Ultimate Strength or Modulus of Rupture S_u or S_r	Elongation Per Cent	Reduction of Area Per Cent
Soft steel from 3½-in. bar	Tension		4	¾	8 and 2	......	21 000	25 000	28 400	55 400	28.9, 50.0	54.3
	Compression		3	¾	2	......	26 800	26 800	27 800	52 800	......	....
	Shear	Solid	8	⅝	2 and 8	......	14 400	16 100	18 100	61 500	......	......
			5	2⅞	5	......	14 200*	15 700	20 400	......	......	......
			3	1¾	5	......	16 700	18 100	19 100	59 200	......	......
		Hollow	6	0.56	2	1/32	11 700	14 900	20 300	......	......	......
			3	2⅞	5	⅛	14 300	16 300	18 100	......	......	......
			3	2⅝	5	3/16	16 500	17 600	18 500	......	......	......
Mild steel from ⅞-in. bar	Tension		4	½	2	......	32 800	32 800	33 600	54 900	43.7	67.9
	Compression		4	¾	2	......	34 800	34 900	36 000	57 500		
	Shear	Solid	4	⅝	2	......	22 900	22 900	23 900	61 800	......	......
		Hollow	4	0.8	2	⅛	21 000	21 000	21 000	57 100	......	......
			4	⅝	2	1/16	21 300	21 300	22 900	......	......	......
			5	0.56	2	1/32	19 700	19 700	21 100	......	......	......
Medium steel from ¾-in. bars	Tension		2	½	2	......	45 000	45 000	45 800	78 000	33.5	57.6
	Compression		2	¾	2	......	41 300	42 800	46 500	81 000	......	......
	Shear	Solid	2	⅝	2	......	31 400	31 700	33 800	79 300	......	......
		Hollow	2	⅝	2	1/16	26 900	27 500	29 500	......	......	......

TABLE 6—(CONTINUED)

AVERAGES OF THE STRENGTHS OBTAINED FROM THE INDIVIDUAL SPECIMENS IN TENSION, IN COMPRESSION, AND IN SHEAR FOR EACH GRADE OF STEEL TESTED

Stress in lb. per sq. in.

Material		Kind of Stress		Number of Tests	Approximate Outside Diameter Inches	Gage Length Inches	Nominal Wall Thickness Inches	Proportional Limit S_p	Useful Limit Point S_e	Yield Point S_y	Ultimate Strength or Modulus of Rupture S_u or S_r	Elongation Per Cent	Reduction of Area Per Cent
Vanadium steel from ⅞-in. bar		Tension		4	½	2	......	52 500	55 900	60 400	109 000	24.0	47.0
		Compression		5	¾	2	......	54 300	57 500	63 100	101 000	......	......
		Shear	Solid	6	⅝	2	......	34 500	37 200	44 600	105 800	......	......
			Hollow	5	0.8	2	⅛	37 500	38 000	42 600	96 000	......	......
				4	⅝	2	¹⁄₁₆	32 100	35 000	42 500	......	......	......
				8	0.56	2	¹⁄₃₂	27 600	30 900	41 900	......	......	......
Nickel steel from slab 2 in. thick	Stress parallel to direction of rolling	Tension		2	½	2	......	37 000	39 500	47 000	87 900	28.3	56.4
		Compression		4	¾	2	......	39 400	42 800	49 100	88 400	......	......
		Shear	Solid	3	⅝	2	......	24 800	26 900	36 600	74 800	......	......
			Hollow	3	⅝	2	¹⁄₁₆	24 300	25 700	35 200	......	......	......
				6	0.56	2	¹⁄₃₂	22 000	24 300	35 200	......	......	......
	Stress perpendicular to direction of rolling	Tension		3	½	2	......	38 800	40 700	47 700	87 700	21.3	39.0
		Compression		5	¾	2	......	38 700	41 300	49 100	92 400	......	......
		Shear	Solid	2	⅝	2	......	26 000	29 000	38 500	77 100	......	......
			Hollow	2	⅝	2	¹⁄₁₆	24 300	27 700	39 200	......	......	......
				4	0.56	2	¹⁄₃₂	20 500	23 500	32 000	......	......	......
Chrome-nickel steel from ¾-in. bars		Tension		6	½	2	......	33 300	34 600	37 300	65 600	37.5	61.5
		Compression		6	¾	2	......	37 600	37 900	39 800	64 300	......	......
		Shear	Solid	4	⅝	2	......	24 100	25 200*	30 900	62 200	......	......
			Hollow	3	⅝	2	¹⁄₁₆	22 800	22 800	24 200	......	......	......

*In error, see Section 9 for discussion.

the fibers do not reach their yield points at the same time. The proportional limit, however, as found from the test of a solid cylindrical torsion specimen has, rather generally, been accepted as the correct shearing proportional limit of the material. Obviously, in the test of a solid cylindrical specimen, it is assumed that the first deviation of the stress-strain diagram from a straight line can be detected at the instant the proportional limit of the outermost fiber has been exceeded. With thin-walled, hollow cylindrical torsion specimens the shearing stress is nearly uniform over the section and the proportional limit represents very closely the correct proportional limit of the material. The results of the tests given in Table 5 show clearly that, although the test of a solid cylindrical torsion specimen is very satisfactory for indicating the general quality or character of the material and its reliability for resisting shear, it is not satisfactory for determining the real elastic shearing strength of the material.

TABLE 7

Relation Between Elastic Shearing Strengths as Found from Hollow and from Solid Torsion Specimens

Material		Ratio $\frac{\text{Shearing Strength, Hollow Specimens}}{\text{Shearing Strength, Solid Specimens}}$		
		Proportional Limit	Useful Limit Point	Yield Point
Soft Steel	Small specimens	.813	.926	1.12*
	Large specimens	.857	.900	.947*
Mild steel		.860	.860	.882
Medium steel		.855†	.868†	.873
Vanadium steel		.800	.830	.940‡
Nickel Steel	Stress parallel to direction of rolling	.886	.904	.962‡
	Stress perpendicular to direction of rolling	.790	.810	.832
Chrome-nickel steel		.946†	.905†	.883

* Yield point of hollow specimens not well defined.

† Probably from 5 to 10 per cent too large because wall thickness of hollow specimens was $\frac{1}{16}$ in. instead of $\frac{1}{32}$ in.

‡ Yield point not well defined for either solid or hollow specimens.

In Table 7 are given values of the ratios of the elastic shearing strength as found from tests of hollow specimens to that found from tests of solid specimens, the elastic strength being indicated by each of the three criteria; namely, proportional limit, useful limit point, and yield point. The results in Tables 6 and 7 justify the following conclusions:

(a) The shearing proportional limit and also the useful limit point obtained from tests with thin-walled hollow cylindrical torsion specimens is eight-tenths to nine-tenths (0.8 to 0.9) of the proportional limit found from solid cylindrical specimens of the same material and a value of eighty-five hundredths (0.85) may be taken with reasonable accuracy for the ratio of the elastic shearing strength found from thin-walled hollow specimens to the similar strength obtained from solid specimens.

(b) The yield point obtained from hollow thin-walled torsion specimens is eighty-five hundredths to nine-tenths (0.85 to 0.9) of the yield point found from solid cylindrical torsion specimens of the same material. This result applies, of course, only to the materials which have a well defined yield point.

The elastic shearing strength (S_p, S_e, and S_y in Table 5) was calculated in each case from the usual formula, $S=\frac{Tc}{J}$ in which T is the twisting moment (inch pounds), J is the polar moment of inertia (inch^4), and c is the radius of the specimen (inch).

By making use of the conclusions stated above a solid cylindrical torsion specimen may be used with considerable confidence to obtain test results from which the true elastic shearing strength may be calculated for steel likely to be used in general structural or machine construction, although the character or development of the elastic breakdown may be obscured in the test of a solid specimen, as is discussed in the next section. The test results from a solid specimen may also be used, of course, to judge of the general properties, quality, and reliability of the material.

The correct value of the ultimate shearing strength of a material can not be obtained, of course, from a torsion test of a solid specimen, although the modulus of rupture, for many purposes, is a satisfactory indication of the general quality of the material and of the shearing resistance against rupture. It is difficult also to determine the ultimate strength from a thin-walled hollow torsion specimen because the specimen fails by collapsing (see Fig. 3). An attempt was made to prevent collapsing by filling the core of the hollow cylindrical specimen with a close (but not tight) fitting rod. It was found, however, that torque was transmitted to the rod. Two rods were then used, one extending in from each end, with somewhat better results. Although the results given in Table 5 on this point are not sufficient for definite conclusions, they indicate that the true ultimate shearing strength is from eight-tenths to nine-tenths (0.8 to 0.9) of the modulus of rupture as obtained from a test of a solid cylindrical torsion specimen.

It will be noted that in the case of the large torsion specimens of soft steel there seems to be some inconsistency in the results as given in Table 5. For instance, the proportional limit and the useful limit point is less for the large solid specimens than for the hollow specimens; these results are contrary to the above conclusions. It was found that binding occurred at the collar of the roller bearing of the stationary head of the torsion machine, particularly at the relatively large twisting moments required for the large solid specimens. This defect was remedied and the diameter of the remaining solid specimens was reduced to $1\frac{3}{4}$ inches so as to use about the same load range on the machine as was used with the hollow specimens. The results, therefore, of the tests with the large solid specimens (diameter $2\frac{7}{8}$ inches) are not considered further.

The test results of all the small torsion specimens and also of the large specimens of soft steel show that the wall thickness must be thin to obtain the correct elastic shearing strength of the material. The wall thickness of the medium carbon steel and of the chrome-nickel steel specimens was 1/16 inch. From the torsion tests of the other materials in which both 1/16-inch and 1/32-inch wall thickness were used, it appears that the correct shearing strength of the medium steel and the chrome-nickel steel is from 5 to 10 per cent lower than that given in Table 6. The ratios as given in Table 7 for these two materials are, therefore, probably somewhat too large.

10. *Characteristics of Elastic Shear Failure.*—Ductile steel rods or specimens from rolled shapes (any specimens from pieces which have been worked considerably in the forming process) when tested in tension, show a rather sudden or abrupt elastic breakdown culminating in a well defined yield point. Steel which has not been worked much, such as the interior material of large rolled bars, does not show such a sudden failure but gives a more gradual curve for the stress-

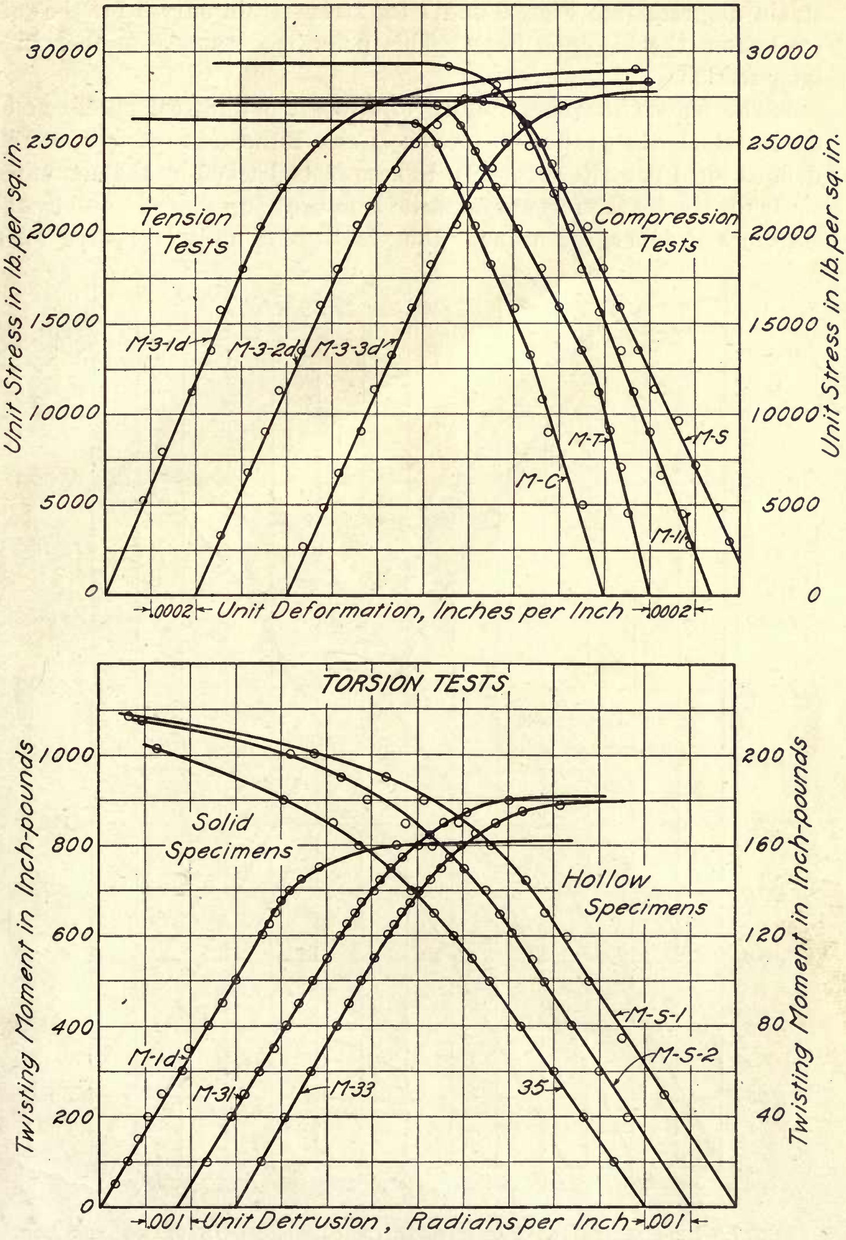

FIG. 6. REPRESENTATIVE STRESS-STRAIN AND MOMENT-STRAIN (TORSION) DIAGRAMS FOR SOFT STEEL—SMALL SPECIMENS

strain diagram (see Figs. 6 and 7 for stress-strain curves for the soft steel from the 3½-inch bar). This point is discussed further in a later section.

The elastic shearing failure of a solid cylindrical specimen of ductile steel when tested in torsion is also rather abrupt with a well defined yield point (see Fig. 8). These facts, backed by the maximum shear theory, by the form of a tensile fracture (cup shaped) and by the slip-lines produced at or near the yield point under repeated load-

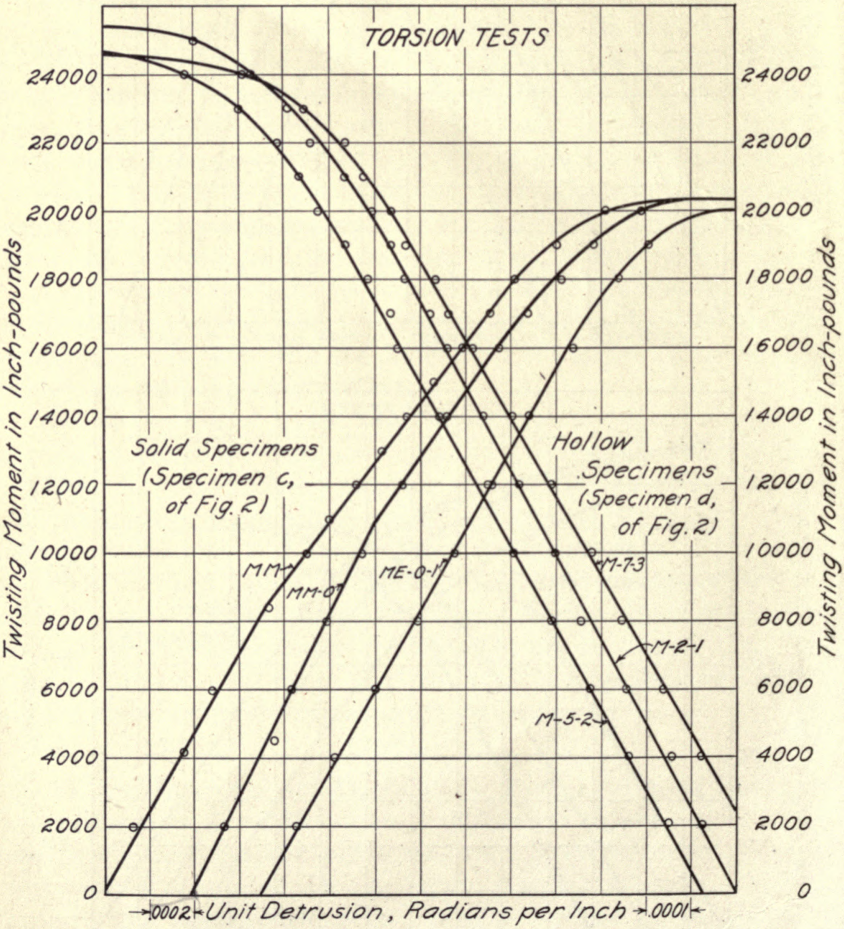

FIG. 7. REPRESENTATIVE MOMENT-STRAIN (TORSION) DIAGRAMS FOR SOFT STEEL—LARGE SPECIMENS

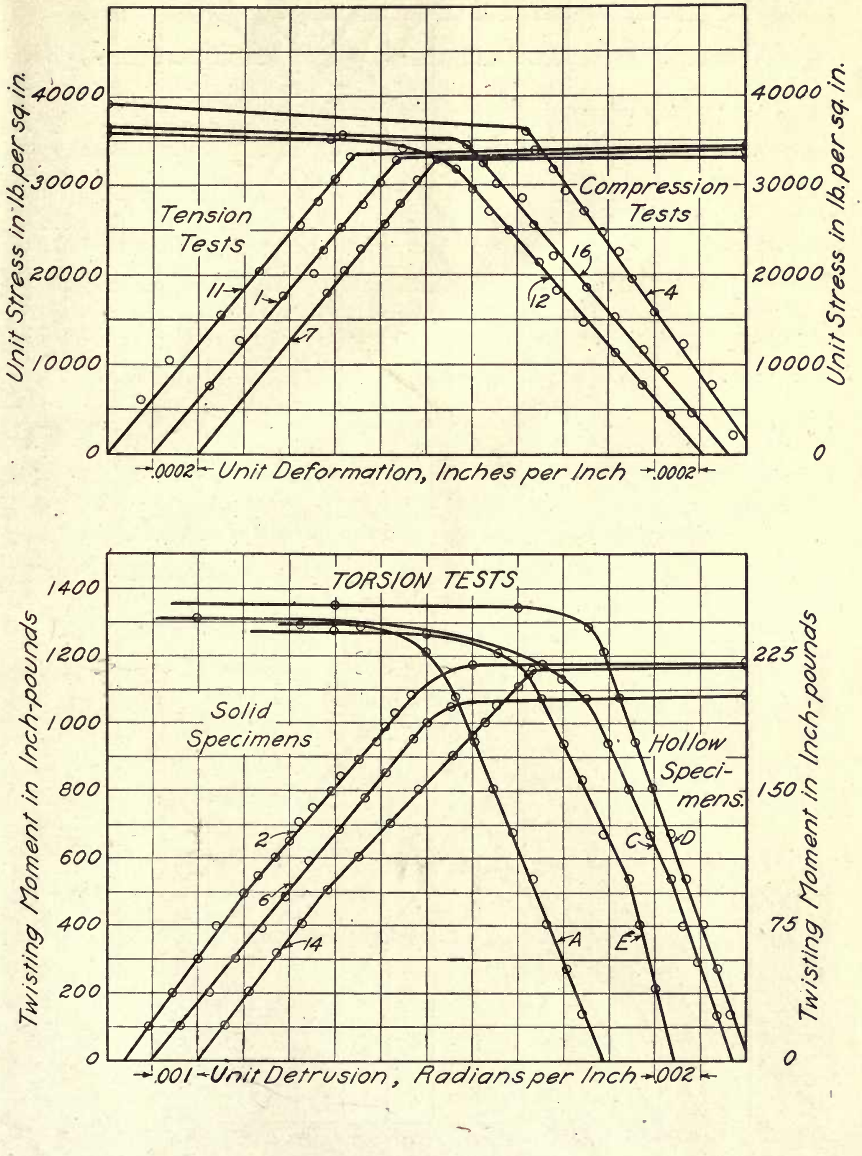

FIG. 8. REPRESENTATIVE STRESS-STRAIN AND MOMENT-STRAIN (TORSION) DIAGRAMS FOR MILD STEEL

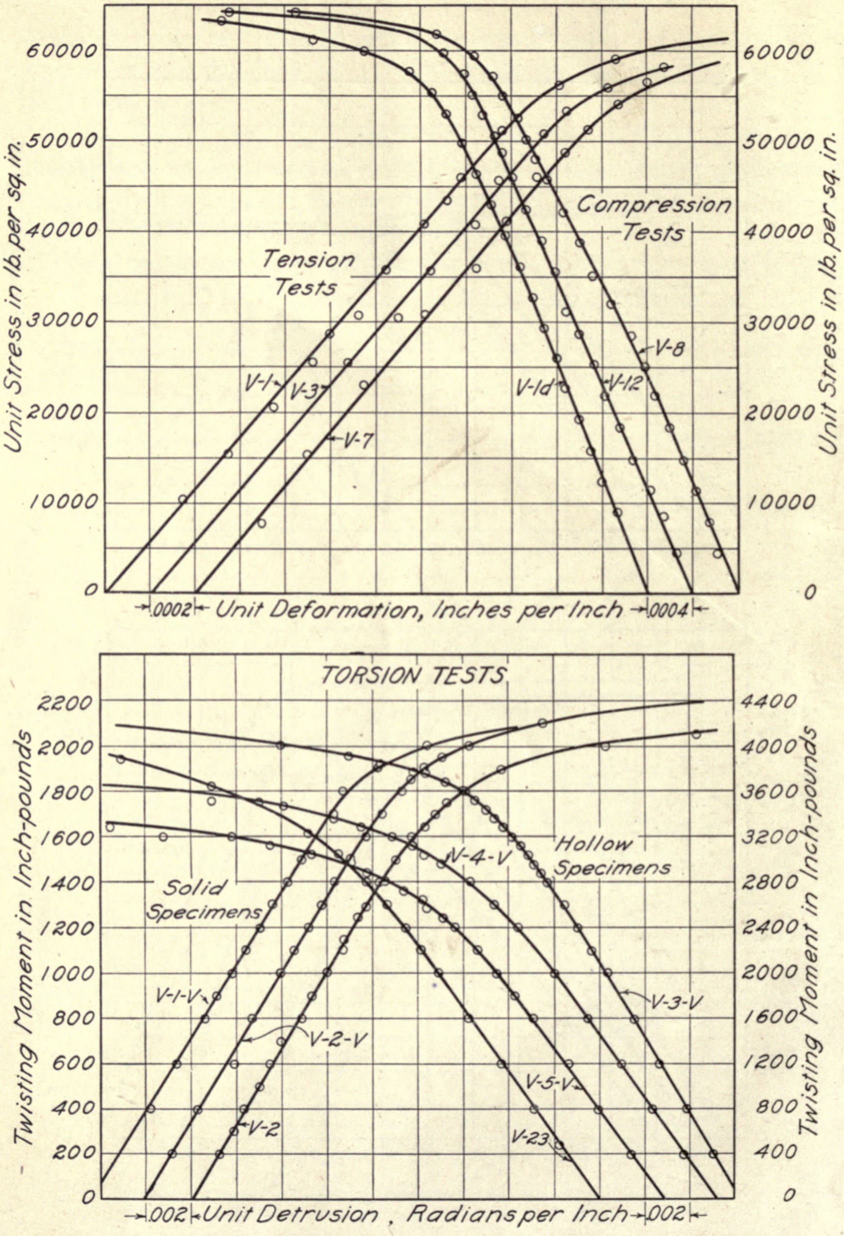

FIG. 9. REPRESENTATIVE STRESS-STRAIN AND MOMENT-STRAIN (TORSION) DIAGRAMS FOR VANADIUM STEEL

ing, have often been used to explain the elastic tensile failure as really a shear failure over an inclined section. And since in a tension test the maximum shearing unit-stress over a section inclined at 45° is one-half of the tensile unit-stress, it has been urged that the elastic shearing strength may best be found from a tension test. Before discussing the experimental data from which the ratios between the elastic shearing and tensile strengths have been found (see next section), it will be well to examine the general character of the elastic shear failure as indicated by the stress-strain diagrams of the thin-walled hollow torsion specimens. From Figs. 6 to 9 which are representative and typical stress-strain diagrams for some of the materials, it will be noted that in most cases the shearing breakdown of elastic action of the various materials is very gradual. In fact, in most cases, it is more gradual than the failure of the elastic action in a tension specimen. It seems difficult, therefore, to account for a tensile elastic breakdown as a failure due to shear, or to have confidence in the use of a tension test for the determination of the elastic shearing strength except for an approximate value. It will also be observed that the solid torsion specimens show a more abrupt elastic failure than a hollow thin-walled specimen of the same material; this fact indicates that the proportional limits of the outer fibers have been somewhat exceeded before a deviation of the stress-strain diagram from a straight line can be detected. This has already been discussed in the section on *Shearing Strength.* The stress-strain diagrams for the medium carbon steel and for the nickel and chrome-nickel steel (not shown) indicate the same characteristics as are shown in Figs. 6 to 9. It appears, therefore, that, contrary to the usual assumption, the shearing breakdown of elastic action of ductile and semi-ductile steel is more gradual than the corresponding tensile or compressive failure.

11. *Ratio of Elastic Shearing Strength to Elastic Tensile Strength.*—Table 8 gives the ratios of the elastic shearing strengths to the elastic tensile strengths for the various materials tested, the elastic strengths being indicated by the proportional limits, the useful limit points, and the yield points. It will be noted from a study of Table 8 that the shearing proportional limits of most of the steels tested are from fifty-five to sixty-five hundredths (0.55 to 0.65) of the tensile proportional limits and that the same statement may be made in the case of the useful limit points. The two materials for which the ratio

TABLE 8

RATIOS OF ELASTIC STRENGTHS IN SHEAR AND TENSION AND ALSO IN TENSION AND COMPRESSION

Material		Ratio $\frac{\text{Shearing Proportional Limit}}{\text{Tensile Proportional Limit}}$		Ratio $\frac{\text{Shearing Useful Limit Point}}{\text{Tensile Useful Limit Point}}$		Ratio $\frac{\text{Shearing Yield Point}}{\text{Tensile Yield Point}}$		Ratio $\frac{\text{Tensile Elastic Strength}}{\text{Compressive Elastic Strength}}$		
		Hollow Specimens	Solid Specimens	Hollow Specimens	Solid Specimens	Hollow Specimens	Solid Specimens	Proportional Limit	Useful Limit Point	Yield Point
Soft steel	Small specimens	.558	.685	.596	.645	.715*	.638	.784	.933	1.04
	Large specimens	.680	.795	.652	.724	.638	.672			
Mild steel		.600	.698	.600	.698	.628	.682	.943	.940	.932
Medium steel		.597†	.698	.611†	.705	.644	.738	1.09‡ or .989	1.05‡ or 1.01	.965
Vanadium steel		.526	.657	.553	.637	.694*	.739	.950	.972	.943
Nickel steel	Stress parallel to direction of rolling	.594	.670	.615	.681	.749	.780	.939	.923	.956
	Stress perpendicular to direction of rolling	.529	.670	.577	.710	.671	.807	1.00	.985	.972
Chrome-nickel steel		.642†	.683	.650	.698	.637	.814	.886	.887	.942

*Yield point of hollow specimens not well defined.

†Probably 5 to 10 per cent too large since hollow specimens with $\frac{1}{16}$ in. wall thickness were used instead of $\frac{1}{32}$ in.

‡Probably in error: only two specimens tested in compression one of which gave very low results (see Table 4).

varies most from the average are the soft steel (large specimens) and the vanadium steel. Owing to the fact that the tension specimens of soft steel were made by quartering a 14-inch length of the 3½-inch bar, they contained much of the inner or "heart" material of the large bar, while the large hollow specimens contained none of the "heart" material. It is doubtful, therefore, whether the tension specimens should be considered as of the same material as the large hollow specimens. Greater variation also was noted in the test results with the soft steel specimens than with the other materials (see section 13 for further discussion). In connection with the vanadium steel it should be stated that due to the greater hardness of the vanadium steel, it was more difficult to produce a smooth hole in drilling out the center of the hollow specimens of this material than of the softer materials. The wall thickness as measured, therefore, may be slightly too large, because the grooves or tool marks were, perhaps, not properly taken into account. If this statement is true, the values of the ratio for vanadium steel are somewhat too small. This statement may also explain the somewhat greater variation in the values of elastic strengths obtained from the thin-walled (1/32 inch thick) vanadium specimens than in those obtained from the corresponding specimens of most of the other materials. It may also account for the greater difference between the results obtained with 1/32-inch and 1/16-inch wall thicknesses for vanadium steel than for the other steels. It is possible, however, that the ratio for vanadium steel is somewhat lower than for the other steels tested.

With these facts in mind it is felt that a study of Table 8 justifies the conclusion that the elastic shearing strength of steel likely to be used in general construction of machines or structures is close to six-tenths (0.6)* of the elastic tensile strength, instead of one-half (0.5) of the elastic tensile strength as assumed in the maximum shear theory of combined stress as expressed by Guest's law. In other words, ductile or semi-ductile steel will not suffer elastic breakdown when the shearing unit-stress developed is one-half of the elastic tensile strength of the material; hence the field in which the maximum strain theory may hold is not as limited as would be the case if the elastic shearing strength were one-half of the tensile strength. The influence of this fact upon the design of machines or structures under combined loading such as thick cylinders, flat plates, crank shafts, girders, etc., can

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*The value of six-tenths is also found by Becker with biaxial loading, see Bulletin No. 85, p. 43, Engineering Experiment Station, University of Illinois.

TABLE 9

RESULTS OF EARLIER EXPERIMENTS

Experiments by Platt and Hayward*		
Materials	Tensile Yield Point† lb. per sq. in.	Ratio $\frac{\text{Torsion, Yield Point}}{\text{Tension, Yield Point}}$
Wrought iron (N. Crown)	33 700	.598
Wrought iron (S. C. Crown)	38 400	.597
Rivet iron	37 700	.615
Siemens Martin steel	37 700	.604
Bessemer steel	69 800	.647
Crucible steel	69 600	.623
Rivet steel	40 000	.571
Cast steel	38 600	.604

Experiments by L. B. Turner‡		
Materials	Tensile Proportional Limit	Ratio $\frac{\text{Torsion, Proportional Limit}}{\text{Tension, Proportional Limit}}$
Mild steel¶ (0.15% C)	31 600	.519
Mild steel§ (0.32% C)	42 300	.577
Tool steel§ (1.25% C)	67 700	.568
Nickel steel§ (3.01% Ni.)	81 200	.503
Experiments by E. L. Hancock**		
Steel tubing	21 000	.500
Nickel steel	76 500	.497
Mild carbon steel	47 000	.649
Steel (Scoble)	64 600	.451
Carbon steel	35 500	.688
Rivet steel	38 900	.602
Nickel steel	56 000	.643
Steel tubing	17 000	.677
Steel tubing	28 000	.576
Steel tubing	20 000	.600

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*Proc. Institution of Civil Engineers, Vol. 90, p. 382, 1888.

†Yield point found by drop of beam.

‡Engineering (London), February 12, 1909, and July 28, 1911.

¶Specimens made of thin-walled tubing.

§Specimens made of small solid rods, 3 in. (tension) and 0.33 in. (torsion).

**Proc. A. S. T. M., p. 376, 1908.

not be discussed here. It makes possible more efficient and economical use of material, under certain conditions, than would be the case if the shearing strength were one-half of the tensile strength.

12. *Results of Earlier Experiments.*—Results of tests of various grades of steel in simple tension and simple torsion made by Platt and Hayward, by L. B. Turner, and by E. L. Hancock are given in Table 9. Platt and Hayward used solid bars 1½ inches in diameter for both tension and torsion tests. The yield point as found by the drop of the beam was used as the elastic strength for both tension and torsion. Two specimens in tension and three in torsion were used for each material. Initial strains were first taken out by repeated loadings. Considering the method of determining the elastic strength and the fact that the elastic strength was of secondary importance in the investigation, the results for the ratios of the elastic strengths in torsion and in tension as found by Platt and Hayward (Table 9) agree well with the values of the same ratio as herein recorded (Table 8).

In the tests by Turner the specimens of mild steel with 0.15 per cent C. were made from annealed weldless steel tubes with an outside diameter of 1 inch and wall thickness of 0.022 inch (No. 24 B.W.G.) for both tension and torsion tests. The wall thickness was not measured for each specimen but an average thickness was determined for twelve short (1 inch) portions cut from three tubes taken at random. The mean thickness was found by first weighing the portions in air and then in water. There was considerable variation in thickness from point to point around a section of a cut portion; in some cases an eccentricity was perceptible to the naked eye. Considerable variation in thickness was shown among the results of the individual specimens. Twenty-one specimens of this particular material were tested.

The specimens for the other three materials tested by Turner were solid ⅜-inch rods for the tension tests and ⅜-inch rods turned down to 0.33-inch diameter for a length of 3½ inches for the torsion tests. The number of specimens was much less than for the steel tubing. All of the specimens were annealed. Nickel steel specimens showed the greatest variation in the results of the individual specimens.

The yielding of all the specimens was very sudden. In all cases the proportional limit and the yield point coincided. This result is quite contrary to the results found in the tests which have already been explained in this bulletin.

When the method of determining the wall thickness of the hollow specimens, the small size of the solid torsion specimens, and the difference in the method of experimenting are considered, the ratios of the elastic strengths in torsion and in tension as found by Turner (Table 9) are in fair agreement with the values of the same ratio as found from the tests herein described (Table 8).

The results of the tests by Hancock show considerable variation. No description of the material is given other than its name in Table 9 and nothing is stated as to the kind or number of specimens used or the method of testing except that the proportional limit was used as a measure of the elastic strength. The results indicate a fair agreement with the results herein recorded for most of the materials, although the low values for the ratio of shear (torsion) to tension for some of the materials is not found by any of the other experimenters.

The values for the ratio of the elastic strengths in tension and in shear as indicated in tests with combined loading have not varied much from 0.6. Becker* found values of 0.59 and 0.62 for two grades of steel tubing. Scoble's† tests are the only ones which indicate that the ratio is less than 0.5 and his criterion of elastic strength and method of experimenting seem somewhat unusual.

A study of the earlier experiments discussed above bear out, in the main, the conclusions already stated for the results presented in this bulletin; namely, that the elastic shearing strength of ductile and semi-ductile steel varies but little from six-tenths (0.6) of the elastic tensile strength.

13. *Ratio of Elastic Tensile Strength to Elastic Compressive Strength.*—The elastic strength in tension and in compression for ductile steels is usually considered to be the same. This assumption is sufficiently accurate for many purposes. In the problems of combined stress and in built-up compression members, however, it is important to know accurately the relation between the elastic tensile and compressive strengths and particularly to know how the strengths are affected by mechanical treatment of the steel such as the amount of working during the rolling process. Tables 6 and 8 indicate that the elastic compressive strength of ductile steel is somewhat greater than the elastic tensile strength except in the case of soft steel. It is clear from a study of Table 8 that the amount of working or rolling has a

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*Bulletin No. 85, Engineering Experiment Station, University of Illinois.

†Philosophical Magazine, May, 1906.

marked effect upon the elastic tensile and compressive strengths of steel. Table 8 indicates that steel which has been rolled into comparatively thin plates or small shapes has an elastic compressive strength approximately 5 per cent greater than the elastic tensile strength whether the elastic strength is judged from the proportional limit, useful limit point, or yield point. In the case of soft steel from the 3½-inch bar which represents material with relatively little working in the rolling process, the compressive proportional limit is much greater (27 per cent) than the tensile proportional limit, while the yield point in compression is somewhat less (4 per cent) than the yield point in tension (see Fig. 6 and Table 8). It appears, therefore, that the criterion of elastic strength may be of much importance, that the tension test so generally used to judge of the elastic properties of steel is not wholly reliable, and that it may be necessary to resort to auxiliary compression tests for material to be used under certain conditions of combined stress and for critical compression members,* at least, until test results and the treatment received by the material can be correlated through more extensive experimental investigations. To what extent the increased compressive strength due to rolling may be attributed to compacting, causing a denser material, or attributed to the arrangement of the crystals of the constituent elements in steel or to other factors is not definitely known. The problem offers opportunity for fruitful experimental work. It was noted that the breakdown of elastic action in compression is fully as gradual as that of the corresponding tensile failure (in most cases more so) except in the case of soft steel. This fact suggests that compacting of steel may be an important factor in determining its strength. The maximum shear theory (Guest's law) of the breakdown of elastic action assumes that the elastic tensile and compressive strengths are the same. The results of Table 8 give further evidence that Guest's law is not an accurate interpretation of the elastic failure of all ductile steel.

14. *Effect of Direction of Rolling.*—It is generally recognized that the direction of rolling of hot-rolled steel influences the grain or

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*In the tests of large built-up columns conducted by Bureau of Standards for the Committee on Steel Columns and Struts of the American Society of Civil Engineers, (see Proc. A. S. C. E. Dec., 1917) it was found that the columns built up of ½-inch to ⅞-inch material showed a considerably lower strength in most cases than those of ⅜-inch material. Compressive tests on specimens sawed from the thick portions showed a decrease in the compressive yield point of 6000 pounds per square inch as compared with the tensile yield point of the same material, while the compressive yield point of specimens sawed from the thin portions was nearly the same as the tensile yield point.

crystal growth or formation, by making the grains or crystals longer in the direction of rolling. It is usually assumed, however, that the elastic strength of the material is not affected by the direction of rolling or, in other words, that the elastic strength is the same in all directions. Some experimental results were obtained on this point in the case of nickel steel. As already stated the specimens of nickel steel were cut from a rolled slab 2 inches thick by 7½ inches wide. From this slab some specimens were cut with their longitudinal axes parallel to the direction of rolling and others with their axes perpendicular to the direction of rolling. Specimens whose axes are parallel to the direction of rolling, when tested in torsion, develop shearing stress perpendicular to the direction of rolling and when tested in tension and compression, of course, develop stress parallel to the direction of rolling, while the reverse is true for the specimens whose axes are perpendicular to the direction of rolling.

The results given in Tables 3, 4, 5, and 6 for nickel steel, although not entirely consistent, fail to indicate any systematic influence of the direction of rolling on the elastic strength of the material either for tension, compression, or shear. However, the ductility as measured by the ultimate tensile elongation and reduction of area shows a marked effect of the direction of rolling. The effect on the ductility is also very noticeable in the appearance of the fracture. While the values given in Table 8 for nickel steel seem to indicate some effect of direction of rolling, a study of the more detailed data in Tables 3 to 6, reveals little if any effect on the elastic strength. The ultimate strengths in tension, in compression, and in shear also show no influence of the direction of rolling. Whether much thinner material such as used for boiler plates, rolled sections, etc., would show more effect of the direction of rolling is not known.

15. *Summary.*—The severe uses to which carbon and alloy steels are put in some phases of engineering, as for example, in automobile and in aeroplane construction, have developed a need for more detailed knowledge of the action of steel, both within and beyond the elastic limit, under various types of stress, as well as of the factors which affect the physical properties of the material. This need, in time, may require some modifications in the tests of the materials and in the methods of interpreting the tests. The following brief summary of the chief points brought out by this investigation are offered as a con-

tribution toward filling part of this need. The conclusions apply to hot-rolled carbon and alloy steels which are representative of those likely to be used in general structural and machine construction.

(1) The correct value of the elastic shearing strength of steel as measured by the proportional limit or the useful limit point may be determined from a torsion test of a hollow thin-walled cylindrical specimen. The correct value of the yield point in shear is also shown in the test of a hollow thin-walled torsion specimen.

(2) The correct value of the elastic shearing strength of steel as measured by the proportional limit or the useful limit point may be taken, with reasonable accuracy, as eighty-five hundredths (0.85) of the elastic strength obtained from a torsion test of a solid cylindrical specimen. The correct value of the yield point for the more ductile materials is slightly more than eighty-five hundredths (0.85) of the yield point obtained from a test of a solid torsion specimen.

(3) A solid cylindrical torsion specimen, therefore, may be used to obtain test results from which the correct value of the elastic shearing strength and the shearing yield point of steel may be calculated by the use of a correction factor, although the character or progress of the breakdown of the elastic action may be obscured in the test of the solid specimen. Test results obtained from a solid specimen may also be used, of course, to judge of the general properties, quality, and reliability of the material.

(4) The correct value of the elastic shearing strength of steel as measured by the proportional limit or the useful limit point is from fifty-five to sixty-five hundredths (0.55 to 0.65) of the elastic tensile strength and may be taken with reasonable accuracy as six-tenths (0.6) of the elastic tensile strength. The maximum shear theory of the failure of elastic action of ductile steel (sometimes called Guest's law) is, therefore, not an accurate statement of the law of elastic breakdown, since Guest's law assumes that the elastic shearing strength is one-half (0.5) of the elastic tensile strength. The maximum shear theory as expressed by Guest's law, however, is of much use in obtaining approximate results.

(5) Contrary to the general belief, the breakdown of the shearing elastic action of ductile and semi-ductile steel as found in these tests is gradual; more gradual, as a rule, than the failure of elastic action in a tension specimen.

(6) The amount of hot-rolling received by steel may have a marked effect upon the relation of the elastic tensile and compressive strengths. For the material from ¾-inch and ⅞-inch bars, the elastic compressive strength is somewhat greater (about 5 per cent) than the elastic tensile strength whether the elastic strength is measured by the proportional limit, useful limit point, or the yield point. For the material from the 3½-inch bar, the compressive proportional limit is very much greater (27 per cent) than the tensile proportional limit, while the compressive yield point is somewhat less (4 per cent) than the tensile yield point.

(7) It appears, therefore, that the choice of the criterion of elastic strength may be of considerable importance in some cases, and that the usual tensile test of material may require supplementary compressive tests or a better correlation between test data and the amount of treatment received during rolling, before the elastic properties of the material as obtained from tensile test data can be relied upon.

(8) The elastic strength and ultimate strength of steel in tension, in compression, and in shear is affected little, if any, by the direction of rolling in the case of a slab 2 inches thick, although the ductility, as measured in tension tests by the percentage of elongation and percentage of reduction of area, is materially less when the stress is perpendicular to the direction of rolling than it is when the stress is parallel to the direction of rolling.

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